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Dark Energy Cosmology with a Rarita-Schwinger Field

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Recent observations of the Cosmic Microwave Background, Supernovae and Sloan Digital Sky Survey (SDSS) show that our universe has a critical energy density, and roughly 2/3 of it is dark energy, which drives the accelerating expansion of the cosmos. In view of the astrophysical data, we find that the equation of state parameter of the dark energy lies in a narrow range around $w = -1$. In this paper, we construct a cosmology model with a Rarita-Schwinger field to realize the equation of state parameter $w < -1$ or $w > -1$ and discuss its stability.

KEY WORDS: cosmology; dark energy; Rarita–Schwinger field.

1. INTRODUCTION

Observations of the Cosmic Microwave Background (CMB) show that the universe is almost flat (Bennett *et al.*, 2003; Netterfield *et al.*, 2002), it follows immediately that our universe has a critical energy density. We know that the baryon makes up of 4% of the critical energy density, and dark matter makes up of 23%. Thus, there must be something makes up of 73% of the critical energy density, which we named dark energy. The astrophysical data of type Ia supernovae reveal that the universe is currently undergoing a period of accelerating expansion (Riess *et al.*, 1998; Tonry *et al.*, 2003). From Einstein equation we can get $\ddot{a}/a = -4\pi/3(\rho + 3p)$. In order to realize $\ddot{a} > 0$, the equation of state w

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of dark energy must satisfy that $w = p/\rho < -1/3$. From above we can get the feature of dark energy: negative pressure that can drive the accelerating expansion of our universe.

Cosmologists have proposed many candidates for dark energy to fit the current observations such as cosmological constant, tachyon (Sen, 2002), quintessence (Caldwell *et al.*, 1998; Gao and Shen, 2002), phantom (Caldwell, 2002) and so on. The major difference among these models are that they predict different equation of state of the dark energy and different history of the cosmos expansion.

Scalar fields have come to play an important role in current models of the dark energy. Quintessence and phantom are based on a scalar field with positive and negative kinetic energy respectively. Quintessence was firstly put forward by Caldwell and Steinhardt. They constructed the model based on a scalar field with positive kinetic energy and can realize the equation of state $-1 < w < -1/3$. However, the analysis of many astrophysics data show that the equation of state $-1 < w$ was not obligatory (Wang and Tegmark, 2004). It is natural to ask what lies on the other side at $w < -1$. To answer this question, Caldwell proposed the phantom model, which is also constructed with a varying scalar field. Phantom has some strange properties. For example, the energy density of it increases with time. It also violates the dominant-energy condition, which helps prohibit time machines and wormholes. However, phantom is an interesting topic because it fits current observations. A striking consequence of dark energy with $w < -1$ is that our universe would end in a "Big Rip" (Caldwell *et al.*, 2003).

In this paper, we construct a dark energy model in terms of a Rarita–Schwinger field, which has been widely studied in other cosmological topics (Christodulakis and Papadopoulos, 1988; Shen *et al.*, 1992). The excellence of this model is that it can realize the equation of state either $w > -1$ or $w < -1$ without switching the sign of the kinetic term in Lagrangian. Therefore, we can unify both quintessence and phantom in the Rarita–Schwinger field model. The presentation of the equation of state w has been derived from the Einstein equation, and the condition for $w < -1$ or $w > -1$ is obtained. We also studied the evolution and attractor property of this model, and show a late time de Sitter attractor, which corresponds to an equation of state $w = -1$ and a cosmic density parameter $\Omega_{rs} = 1$ by choosing an exponential potential. The big rip is also discussed in our model.

2. THE MODEL

The flat Robertson–Walker metric is given by

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) \tag{2.1}$$

where $a\,(t)$ is the scale factor of the universe.

Since the space is homogeneous, the Einstein field equations require the Rarita–Schwinger field to be a function of the time only. We choose $\psi_m = \psi_m(t)$ and $\gamma\psi \equiv \gamma^i\psi_i = 0 (i = 1, 2, 3)$. Thus, the lagrangian of the Rarita–Schwinger field takes the very simple form

$$L = \sqrt{-g}\left[-\frac{i}{2}(\psi_i^+\dot{\psi}_i - \dot{\psi}_i^+\psi_i) - V\right] \tag{2.2}$$

The variation of the lagrangian yields the equations of motion of the field

$$i\dot{\psi}_i + \frac{3}{2}i\frac{\dot{a}}{a}\psi_i + \frac{\partial V}{\partial \psi_i^+} = 0 \tag{2.3}$$

$$i\dot{\psi}_i^+ + \frac{3}{2}i\frac{\dot{a}}{a}\psi_i^+ - \frac{\partial V}{\partial \psi_i} = 0 \tag{2.4}$$

From the above equations, we can obtain

$$(\dot{\psi_i^+\psi_i}) + 3H\psi_i^+\psi_i = 0 \tag{2.5}$$

Thus, in an expanding universe $\psi_i^+\psi_i$ decays as fast as $1/a^3$. However, it does not imply that the energy density of the Rarita–Schwinger field follows the same behavior.

The Einstein field equations in this model are

$$3\frac{\dot{a}^2}{a^2} = \frac{\kappa^2}{3}(V + \rho_\gamma), \tag{2.6}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = \frac{\kappa^2}{3}\left[V + \frac{i}{2}(\psi_i^+\dot{\psi}_i - \dot{\psi}_i^+\psi_i) - P_\gamma\right]. \tag{2.7}$$

where ρ_γ is the density of barotropic fluid with a equation of state $P_\gamma = (\gamma - 1)\rho_\gamma$. The Rarita–Schwinger field contributes the energy density ρ_{rs} and pressure P_{rs} as follows

$$\rho_{rs} = V \tag{2.8}$$

$$P_{rs} = -V - \frac{i}{2}(\psi_i^+\dot{\psi}_i - \dot{\psi}_i^+\psi_i) \tag{2.9}$$

The equation of state of the spinor is

$$w = \frac{P_{rs}}{\rho_{rs}} = \frac{-V - \frac{i}{2}(\psi_i^+\dot{\psi}_i - \dot{\psi}_i^+\psi_i)}{V}. \tag{2.10}$$

Considering (2.3) and (2.4), we can rewrite (2.10) as

$$w = \frac{P_{sp}}{\rho_{sp}} = \frac{-V + V'\psi_i^+\psi_i}{V}. \tag{2.11}$$

where a prime ($'$) denotes a derivative with respect to $\psi_i^+\psi_i$.

The equation of state w of the Rarita–Schwinger field can be either larger than -1 or smaller than -1 under different conditions. Under the condition $V > 0$, when $V'\psi_i^+\psi_i > 0$, then $w > -1$; and $w < -1$ while $V'\psi_i^+\psi_i < 0$. Thus, the Rarita–Schwinger field can be used instead of quintessence or phantom by choosing different form of potential without switching the sign of the kinetic term in Lagrangian.

3. EVOLUTION AND STABILITY OF THE MODEL

Now we discuss the stability of this model, to do so, we rewrite the Einstein equations as

$$\dot{H} = -\frac{\kappa^2}{2}(\rho_\gamma + P_\gamma + V'\psi_i^+\psi_i) \tag{3.1}$$

$$\dot{\rho_\gamma} = -3H(\rho_\gamma + P_\gamma) \tag{3.2}$$

Introduce new variables as follows:

$$x = \frac{\kappa\psi_i^+\psi_i}{\sqrt{3}H^2}, \tag{3.3}$$

$$y = \frac{\kappa\sqrt{V}}{\sqrt{3}H}, \tag{3.4}$$

$$z = \kappa H \tag{3.5}$$

$$N = \ln a. \tag{3.6}$$

We choose a specific potential to discuss the evolution of the equation of state and the cosmic density parameter of the Rarita–Schwinger field. We consider the exponential potential, which has been widely investigated.

$$V(\psi_i^+, \psi_i) = V_0 e^{-\lambda\kappa^3\psi_i^+\psi_i} \tag{3.7}$$

Thus, the system becomes

$$\frac{dx}{dN} = -3x + \gamma x(1 - y^2) - 3\sqrt{3}\lambda x^2 y^2 z^2, \tag{3.8}$$

$$\frac{dy}{dN} = \frac{3\sqrt{3}}{2}\lambda x y z^2 + \frac{3}{2}\gamma y(1 - y^2) - \frac{3\sqrt{3}}{2}\lambda x y^3 z^2, \tag{3.9}$$

$$\frac{dz}{dN} = -\frac{3}{2}z\gamma(1 - y^2) + \frac{3\sqrt{3}}{2}\lambda x y^2 z^3. \tag{3.10}$$

Also we have a constraint equation

$$\frac{\kappa^2\rho_\gamma}{3H^2} + y^2 = 1. \tag{3.11}$$

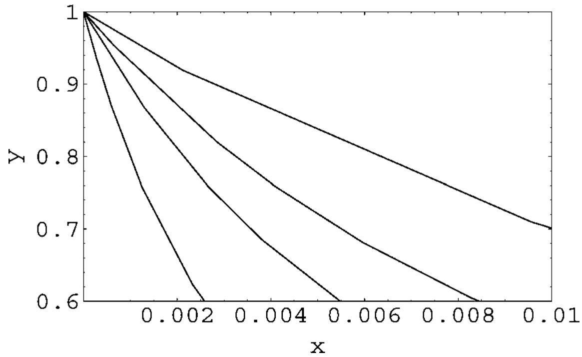

Fig. 1. Phase plane for different initial x, y and z. We choose $\lambda = 0.0001, \gamma = 1$.

In the following, we will investigate a physical meaningful solution and discuss its stability. We can get critical points by setting the right hand of the equations (3.8) (3.9) (3.10) to zero. So we get a meaningful point (x_c, y_c, z_c) of the system as $(0, 1, z_c)$ where z_c could be any constant, which corresponding to a de Sitter phase. In this case, the equation of state would evolute to -1 and the cosmic energy density parameter of the Rarita–Schwinger field would rise to 1. We write the variables near the critical point (x_c, y_c, z_c) in the form $x = x_c + u$, $y = y_c + v$, $z = z_c + \delta$ with u, v, δ the perturbations of the variables near the critical points. Substitute the expression to equations (3.8) (3.9) (3.10) we can get a matrix of the perturbations

$$\begin{pmatrix} -3 & 0 & 0 \\ 0 & -3\gamma & 0 \\ \frac{3\sqrt{3}}{2} z_c^3 \lambda & 3 z_c \gamma & 0 \end{pmatrix} \tag{3.12}$$

whose eigenvalues determine the stability of the critical points. In our de Sitter case, the eigenvalues are $(-3, -3\gamma, 0)$, which indicate the critical point is a dynamical attractor. We can get some insight into the property of the system by drawing a phrase plane Fig. 1.

The numerically study results of the equation of state and the cosmic density parameter in our model are shown in Figs. 2 and 3.

As pointed by Caldwell *et al.* (1998) the scalar factor will blow up in a finite proper time with a constant negative value of w. However, in our model the equation of state w would approach to -1 dramatically and then in principle the big rip can be avoided.

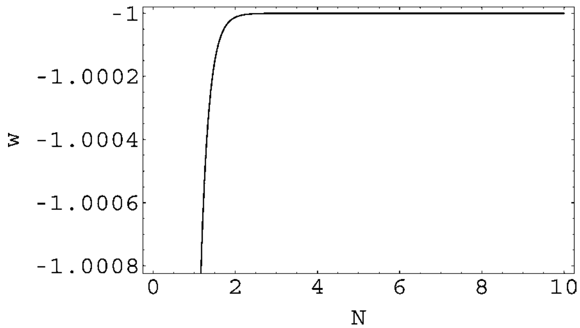

Fig. 2. The equation of state of the Rarita–Schwinger field by choosing $\lambda = 0.0001$, $\gamma = 1$.

4. CONCLUSIONS

In conclusion, we show that the Rarita–Schwinger field could play the role of dark energy in driving the accelerating universe, and it can realize the equation-of-state $w < -1$ or $w > -1$ under different condition without switching the sign of the kinetic term. We find a physical meaningful critical point which corresponding to the de Sitter phase and prove it is a dynamical attractor by choosing an exponential potential. The evolution of the equation of state and the cosmic density parameter of the Rarita–Schwinger field have also been discussed by numerical analysis. Finally, we show that in our model the the big rip can be avoided. The most important excellence of this model is that it can realize $w < -1$ without negative kinetic energy and fit the current observations.

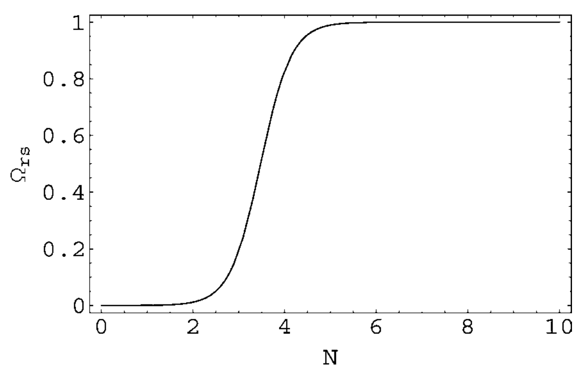

Fig. 3. The cosmic density parameter of the Rarita–Schwinger field by choosing $\lambda = 0.0001$, $\gamma = 1$.

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